

WIDTH DIFFERENCE IN THE $D^0 - \bar{D}^0$ SYSTEM^a

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The motivation most often cited in searches for $D^0 - \bar{D}^0$ mixing lies with the possibility of observing a signal from new physics which dominates that from the Standard Model. We discuss recent theoretical and experimental results in $D^0 - \bar{D}^0$ mixing, including new experimental measurements from CLEO and FOCUS collaborations and their interpretations.

1 Introduction

Neutral meson-antimeson mixing provides important information about electroweak symmetry breaking and quark dynamics. In that respect, the $D^0 - \bar{D}^0$ system is unique as it is the only system that is sensitive to the dynamics of the bottom-type quarks. The $D^0 - \bar{D}^0$ mixing proceeds extremely slowly, which in the Standard Model (SM)^{1,2,3,4,5} is usually attributed to the absence of superheavy quarks destroying GIM cancelations. This feature makes it sensitive both to physics beyond the Standard Model and to long-distance QCD effects.

The low energy effect of new physics particles can be naturally written in terms of a series of local operators of increasing dimension generating $\Delta C = 2$ transitions. These operators, along with the Standard Model contributions, generate the mass and width splittings for the eigenstates of $D^0 - \bar{D}^0$ mixing matrix defined as

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|\bar{D}^0\rangle \quad , \quad (1)$$

with complex parameters p and q determined from the phenomenological (CPT-invariant) $D^0 - \bar{D}^0$ mass matrix⁶. It is convenient to normalize the mass and

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width differences to define two dimensionless variables x and y

$$x \equiv \frac{m_2 - m_1}{\Gamma}, \quad y \equiv \frac{\Gamma_2 - \Gamma_1}{2\Gamma}. \quad (2)$$

where $m_i(\Gamma_i)$ is a mass (width) of the corresponding state, $D_{\frac{1}{2}}$. Clearly, y is built from the decays of D into the physical states, and so it should be dominated by the SM contributions. If CP-violation is neglected, then $p = q$ and $|D_{\frac{1}{2}}\rangle$ become eigenstates of CP . To set up a relevant formalism, let us recall that in perturbation theory, the ij^{th} element of the $D^0 - \bar{D}^0$ mass matrix can be represented as

$$\left[M - i\frac{\Gamma}{2} \right]_{ij} = \frac{1}{2m_D} \langle D_i^0 | \mathcal{H}_W^{\Delta C=2} | D_j^0 \rangle + \frac{1}{2m_D} \sum_I \frac{\langle D_i^0 | \mathcal{H}_W^{\Delta C=1} | I \rangle \langle I | \mathcal{H}_W^{\Delta C=1\dagger} | D_j^0 \rangle}{m_D^2 - E_I^2 + i\epsilon}. \quad (3)$$

Here the first term of Eq. (3) comes from the local $\Delta C = 2$ (box and dipenguin) operators. These contributions affect ΔM only and expected to be small in the Standard Model^{3,4,5}. It is therefore natural to expect that the $\Delta C = 2$ part of Eq. (3) might receive contributions from the effective operators generated by the new physics interactions. Next come the bilocal contributions which are induced by the insertion of two Hamiltonians changing the charm quantum number by one unit, i.e. built out of $\Delta C = 1$ operators. This class of terms contributes to both x and y and is believed to give the dominant SM contribution to the mixing due to various nonperturbative effects. Some enhancement due to the $\Delta C = 1$ operators induced by new physics is also possible, but unlikely given the strong experimental constraints provided by the data on D meson decays. Yet, the motivation most often cited in searches for $D^0 - \bar{D}^0$ mixing lies with the possibility of observing a signal from new physics which dominates that from the Standard Model. It is therefore extremely important to estimate the Standard Model contribution to x and y .

The mass and width differences x and y can be measured in a variety of ways, for instance in semileptonic $D \rightarrow Kl\nu$ or nonleptonic $D \rightarrow KK$ or $D \rightarrow K\pi$ decays. Let us define the D meson decay amplitudes into a final state f as

$$A_f \equiv \langle f | \mathcal{H}_W^{\Delta C=1} | D^0 \rangle, \quad \bar{A}_f \equiv \langle f | \mathcal{H}_W^{\Delta C=1} | \bar{D}^0 \rangle. \quad (4)$$

It is also useful to define the complex parameter λ_f :

$$\lambda_f \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f}. \quad (5)$$

Let us first consider the processes that are relevant to the FOCUS⁷ and CLEO⁸ experiments. Those are the doubly-Cabibbo-suppressed $D^0 \rightarrow K^+\pi^-$ decay, the singly-Cabibbo-suppressed $D^0 \rightarrow K^+K^-$ decay, the Cabibbo-favored $D^0 \rightarrow K^-\pi^+$ decay, and the three CP-conjugate decay processes. Let us write down approximate expressions for the time-dependent decay rates that are valid for times $t < 1/\Gamma$. We take into account the experimental information that x , y and $\tan\theta_c$ are small, and expand each of the rates only to the order that is relevant to the CLEO and FOCUS measurements:

$$\begin{aligned}
\Gamma[D^0(t) \rightarrow K^+\pi^-] &= e^{-\Gamma t} |\bar{A}_{K^+\pi^-}|^2 |q/p|^2 \\
&\times \left\{ |\lambda_{K^+\pi^-}^{-1}|^2 + [\Re(\lambda_{K^+\pi^-}^{-1})y + \Im(\lambda_{K^+\pi^-}^{-1})x]\Gamma t + \frac{1}{4}(y^2 + x^2)(\Gamma t)^2 \right\}, \\
\Gamma[\bar{D}^0(t) \rightarrow K^-\pi^+] &= e^{-\Gamma t} |A_{K^-\pi^+}|^2 |p/q|^2 \\
&\times \left\{ |\lambda_{K^-\pi^+}|^2 + [\Re(\lambda_{K^-\pi^+})y + \Im(\lambda_{K^-\pi^+})x]\Gamma t + \frac{1}{4}(y^2 + x^2)(\Gamma t)^2 \right\}, \quad (6) \\
\Gamma[D^0(t) \rightarrow K^+K^-] &= e^{-\Gamma t} |A_{K^+K^-}|^2 \{1 + [\Re(\lambda_{K^+K^-})y - \Im(\lambda_{K^+K^-})x]\Gamma t\}, \\
\Gamma[\bar{D}^0(t) \rightarrow K^+K^-] &= e^{-\Gamma t} |\bar{A}_{K^+K^-}|^2 \{1 + [\Re(\lambda_{K^+K^-}^{-1})y - \Im(\lambda_{K^+K^-}^{-1})x]\Gamma t\}, \\
\Gamma[D^0(t) \rightarrow K^-\pi^+] &= e^{-\Gamma t} |A_{K^-\pi^+}|^2, \quad \Gamma[\bar{D}^0(t) \rightarrow K^+\pi^-] = e^{-\Gamma t} |\bar{A}_{K^+\pi^-}|^2.
\end{aligned}$$

Within the Standard Model, the physics of $D^0 - \bar{D}^0$ mixing and of the tree level decays is dominated by the first two generations and, consequently, CP violation can be safely neglected. In all ‘reasonable’ extensions of the Standard Model, the six decay modes of Eq. (6), are still dominated by the Standard Model CP conserving contributions. On the other hand, there could be new short distance, possibly CP violating contributions to the mixing amplitude M_{12} . Allowing for only such effects of new physics, the picture of CP violation is simplified since there is no direct CP violation. The effects of indirect CP violation can be parameterized in the following way

$$\begin{aligned}
|q/p| &= R_m, \\
\lambda_{K^+\pi^-}^{-1} &= \sqrt{R} R_m^{-1} e^{-i(\delta+\phi)}, \\
\lambda_{K^-\pi^+} &= \sqrt{R} R_m e^{-i(\delta-\phi)}, \\
\lambda_{K^+K^-} &= -R_m e^{i\phi}.
\end{aligned} \quad (7)$$

Here R and R_m are real and positive dimensionless numbers. CP violation in mixing is related to $R_m \neq 1$ while CP violation in the interference of decays with and without mixing is related to $\sin\phi \neq 0$. The choice of phases and signs in Eq. (7) is consistent with having the weak phase difference $\phi = 0$ in the Standard Model and the strong phase difference $\delta = 0$ in the $SU(3)$

limit. The weak phase ϕ is universal for $K\pi$ and KK final states under our assumption of negligible direct CP violation. We further define

$$\begin{aligned} x' &\equiv x \cos \delta + y \sin \delta, \\ y' &\equiv y \cos \delta - x \sin \delta. \end{aligned} \quad (8)$$

With the assumption that there is no direct CP violation in the processes that we study, and using the parameterizations (7) and (8), we can rewrite Eqs. (6) as follows:

$$\begin{aligned} \Gamma[D^0(t) \rightarrow K^+\pi^-] &= e^{-\Gamma t} |A_{K^-\pi^+}|^2 \\ &\times \left[R + \sqrt{R} R_m (y' \cos \phi - x' \sin \phi) \Gamma t + \frac{R_m^2}{4} (y^2 + x^2) (\Gamma t)^2 \right], \\ \Gamma[\overline{D}^0(t) \rightarrow K^-\pi^+] &= e^{-\Gamma t} |A_{K^-\pi^+}|^2 \\ &\times \left[R + \sqrt{R} R_m^{-1} (y' \cos \phi + x' \sin \phi) \Gamma t + \frac{R_m^{-2}}{4} (y^2 + x^2) (\Gamma t)^2 \right] \\ \Gamma[D^0(t) \rightarrow K^+K^-] &= e^{-\Gamma t} |A_{K^+K^-}|^2 [1 - R_m (y \cos \phi - x \sin \phi) \Gamma t], \quad (9) \\ \Gamma[\overline{D}^0(t) \rightarrow K^+K^-] &= e^{-\Gamma t} |A_{K^+K^-}|^2 [1 - R_m^{-1} (y \cos \phi + x \sin \phi) \Gamma t], \\ \Gamma[D^0(t) \rightarrow K^-\pi^+] &= \Gamma[\overline{D}^0(t) \rightarrow K^+\pi^-] = e^{-\Gamma t} |A_{K^-\pi^+}|^2. \end{aligned}$$

By studying various combinations of these modes we can pin down the values of x and y in $D^0 - \overline{D}^0$ system.

2 Theoretical expectations

The leading piece of the short-distance part of the mixing amplitude is known to be small^{1,2,3,4,5}, but it is instructive to see why it is so. We will also complement the discussion by including leading $1/m_c$ corrections.

As discussed above, the lifetime difference is associated with the long-distance contribution to Eq. (3), i.e. the double insertion of $\Delta C = 1$ effective Hamiltonian

$$\mathcal{H}_W^{\Delta C=1} = -\frac{G_F}{\sqrt{2}} \sum_q \xi_q \{ C_1(\mu) \bar{u}_\alpha \Gamma_\mu q_\beta \bar{q}_\beta \Gamma^\mu c_\alpha + C_2(\mu) \bar{u}_\alpha \Gamma_\mu q_\alpha \bar{q}_\beta \Gamma^\mu c_\beta \} \quad (10)$$

where $\Gamma_\mu = \gamma_\mu(1 + \gamma_5)$ and $\xi_q = V_{cq}^* V_{uq}$ represents the appropriate CKM factor for $\psi = d, s$. $C_1(m_c) \simeq -0.514$ and $C_2(m_c) \simeq 1.270$, as found in a NLO QCD calculation with ‘scheme-independent’ prescription. Hereafter we shall not write the scale dependence of Wilson coefficients explicitly. The width difference y can be written as an imaginary part of the matrix element of the

time-ordered product of two $\Delta C = 1$ Hamiltonians of Eq. (10). Physically, it is generated by a set of on-shell intermediate states, and therefore, constitutes an intrinsically non-local quantity. However, in the limit $m_c/\Lambda_{QCD} \rightarrow \infty$ the momentum flowing through the light (s and d quark) degrees of freedom is large and an Operator Product Expansion (OPE) can be performed. As a result, both x and y can be represented by a series of matrix elements of *local* operators of increasing dimension. In other words, if a typical hadronic distance $z \gg 1/m_c$, then the decay is a local process. Of course, significant corrections to the leading term of this series are expected, as the expansion parameter Λ/m_c ($\Lambda \sim \Lambda_{QCD}$ is some hadronic parameter) is not small.

It is well known that y should vanish in the limit of equal quark masses by the virtue of GIM cancelation mechanism. For the $D\bar{D}$ system it is equivalent to the requirement of flavor $SU(3)$ symmetry. The question here is by how much $SU(3)$ is broken. The (parametrically) leading contribution to x and y comes from the matrix elements of operators of dimension six

$$\begin{aligned} O_1 &= \bar{u}\gamma_\mu(1+\gamma_5)c\bar{u}\gamma_\mu(1+\gamma_5)c, & O'_1 &= \bar{u}(1-\gamma_5)c\bar{u}(1-\gamma_5)c \\ O_2 &= \bar{u}_i\gamma_\mu(1+\gamma_5)c_k\bar{u}_k\gamma_\mu(1+\gamma_5)c_i, & O'_2 &= \bar{u}_i(1-\gamma_5)c_k\bar{u}_k(1-\gamma_5)c_i \end{aligned} \quad (11)$$

Using Fierz identities and performing necessary integrations we obtain

$$\begin{aligned} \Delta\Gamma_D^{(6)} &= \frac{N_c+1}{\pi N_c} X_D \frac{(m_s^2 - m_d^2)^2}{m_c^2} \frac{m_s^2 + m_d^2}{m_c^2} \left[C_2^2 + 2C_1C_2 + C_1^2 N_c \right. \\ &\quad \left. - \frac{2(2N_c-1)}{1+N_c} \frac{B'_D}{B_D} \frac{M_D^2}{(m_c+m_u)^2} \left(C_2^2 + \frac{2-N_c}{2N_c-1} (C_1^2 N_c + 2C_1C_2) \right) \right] \end{aligned} \quad (12)$$

with $N_c = 3$ being the number of colors. This result was reported in ¹⁰. Numerically, the effect of including QCD evolution amounts to the enhancement of the box diagram estimate by approximately a factor of two. As one can easily see, a standard box diagram contribution is recovered in the limit $C_1 \rightarrow 0$, $C_2 \rightarrow 1$ where the QCD evolution is turned off

$$\begin{aligned} \Delta m_D^{\text{box}} &= \frac{2}{3\pi^2} X_D \frac{(m_s^2 - m_d^2)^2}{m_c^2} \left[1 - \frac{5}{4} \frac{B'_D}{B_D} \frac{M_D^2}{(m_c+m_u)^2} \right], \\ \Delta\Gamma_D^{\text{box}} &= \frac{4}{3\pi} X_D \frac{(m_s^2 - m_d^2)^2}{m_c^2} \frac{m_s^2 + m_d^2}{m_c^2} \left[1 - \frac{5}{2} \frac{B'_D}{B_D} \frac{M_D^2}{(m_c+m_u)^2} \right], \end{aligned} \quad (13)$$

with X_D is given by $X_D \equiv \xi_s \xi_d B_D G_F^2 M_D F_D^2$. Also, the B-parameters $B_D = B'_D = 1$ in the usual vacuum saturation approximation to

$$\langle D^0 | O_1 | \bar{D}^0 \rangle = \left(1 + \frac{1}{N_c} \right) \frac{4F_D^2 m_D^2}{2m_D} B_D,$$

$$\begin{aligned}
\langle D^0 | O'_1 | \bar{D}^0 \rangle &= - \left(1 - \frac{1}{2N_c} \right) \frac{4m_D^2}{(m_c + m_u)^2} \frac{F_D^2 m_D^2}{2m_D} B'_D, \\
\langle D^0 | O_2 | \bar{D}^0 \rangle &= \left(1 + \frac{1}{N_c} \right) \frac{4F_D^2 m_D^2}{2m_D} B_D, \\
\langle D^0 | O'_2 | \bar{D}^0 \rangle &= - \left(\frac{1}{N_c} - \frac{1}{2} \right) \frac{4m_D^2}{(m_c + m_u)^2} \frac{F_D^2 m_D^2}{2m_D} B'_D,
\end{aligned} \tag{14}$$

where $2m_D$ in the denominator comes from the normalization of meson states and F_D is a D -meson decay constant. It is clear from Eq. (12) that the smallness of the leading order result comes from the factor of $(m_s^2 - m_d^2)^2/m_c^2$ which represents the GIM cancelation among the intermediate s and d quark states and from the factor $(m_s^2 + m_d^2)/m_c^2$ which represents the helicity suppression of the intermediate state quarks. At the end, $y \ll x \ll 0.1\%$.

Of course, one should be concerned with the size of (parametrically suppressed) corrections to Eq. (12). This is especially important for the calculation of y because of the $SU(3)$ and helicity suppression of the parametrically leading term. For example, perturbative QCD corrections, while suppressed by $\alpha_s(m_s)$, include the gluon emission diagrams, which do not exhibit helicity suppression factors of m_s^2 .

In addition, both $SU(3)$ and helicity suppression factors $(m_s^2 - m_d^2)^2(m_s^2 + m_d^2)$ can be lifted at higher orders in Λ/m_c , which calls for a certain reorganization of the operator expansion. In spite of being parametrically suppressed, those “corrections” are in fact numerically larger than the leading order term. It was realized^{1,11} that the higher order contributions from the operators of dimension nine and twelve that represent interactions with the background quark condensates do exactly that.

Taking into account new operator structures generated by the renormalization group running of the effective Hamiltonian from M_W down to m_c , the contribution of dimension nine operators reads

$$\begin{aligned}
\Delta M_D^{(9)} &= 4\xi_s^2 G_F^2 \frac{m_s^2 - m_d^2}{N_c m_c^3} v^\alpha \left\{ (N_c C_1^2 + 2C_1 C_2 + C_2^2) \right. \\
&\times \left[\langle D^0 | (\bar{u}\Gamma_\alpha c)(\bar{u}\Gamma_\mu c)(\bar{\psi}\Gamma^\mu \psi) | \bar{D}^0 \rangle + \text{others} \right] \\
&+ 2C_2^2 \left[\langle D^0 | (\bar{u}\Gamma_\alpha T^a c)(\bar{u}\Gamma_\mu T^a c)(\bar{\psi}\Gamma^\mu \psi) | \bar{D}^0 \rangle + \langle D^0 | (\bar{u}\Gamma_\alpha c)(\bar{u}\Gamma_\mu T^a c)(\bar{\psi}\Gamma^\mu T^a \psi) | \bar{D}^0 \rangle \right. \\
&+ \left. \left. \langle D^0 | (\bar{u}\Gamma_\alpha T^a c)(\bar{u}\Gamma_\mu c)(\bar{\psi}\Gamma^\mu T^a \psi) | \bar{D}^0 \rangle + \text{others} \right] \right\} \tag{15}
\end{aligned}$$

$$\begin{aligned}
& + 2N_c C_2^2 (d^{abc} + i f^{abc}) \left[\langle D^0 | (\bar{u} \Gamma_\alpha T^a c) (\bar{u} \Gamma_\mu T^b c) (\bar{\psi} \Gamma^\mu T^c \psi) | \bar{D}^0 \rangle + \text{others} \right] \\
& + 4N_c C_1 C_2 \left[\langle D^0 | (\bar{u} \Gamma_\alpha c) (\bar{u} \Gamma_\mu T^a c) (\bar{\psi} \Gamma^\mu T^a \psi) | \bar{D}^0 \rangle + \text{others} \right] \Big\}
\end{aligned}$$

Here $\bar{\psi} \Gamma^\mu \psi = (\bar{s} \Gamma^\mu s - \bar{d} \Gamma^\mu d)$ and ‘others’ denotes operators with cyclic permutations of α, μ, μ indices. v represents the heavy quark velocity. Naive power counting argument of Ref. ¹ implies that the U -spin violating operator $\bar{\psi} \Gamma^\mu \psi$ would scale like $m_s \Lambda^2$ and therefore, the overall contribution to x and y is multiplied by a factor of m_s^3 , compared to the leading term, where $x \sim m_s^4$ and $y \sim m_s^6$. In order to develop an imaginary part (and so generate y), a gluon correction should be considered. Therefore, the contribution of dimension nine operators to y is suppressed by both α_s and phase space factors compared to x , $y^{(9)} \sim (\alpha_s/16\pi)x^{(9)} \ll x^{(9)}$. While it is impossible to estimate this contribution reliably (there are unknown matrix elements of 15 operators), naive power counting rules imply that it dominates the parametrically leading terms in the expansion of x^1 and y^{11} .

The next important contribution to y is obtained at the next order in $1/m_c$ and is given by a subset of matrix elements of the operators of dimension twelve. This contribution is obtained by cutting all light fermion lines and adding a gluon to transfer large momentum. It is therefore represented by a set of eight-fermion operators. While suppressed by α_s/m_c^2 , it again lifts another factor of m_s . More importantly, $y^{(12)} \sim x^{(12)}$! This observation ¹¹ comes from the fact that imaginary part of the diagram that is needed for generating $\Delta\Gamma_D$ can also be obtained by dressing the gluon propagator by quark and gluon “bubbles”. The resulting $\alpha_s(m_c)$ suppression is largely compensated by the “enhancement” from the QCD β function. This results in the estimate ^{11,12}

$$x, y \sim 0.1\%, \quad (16)$$

which is obtained from the naive dimensional analysis, as there are too many unknown matrix elements for the accurate prediction to be made.

Indeed, the short-distance analysis, while systematic, is valid as long as one believes that the charmed quark is sufficiently heavy for $1/m_c$ expansion to be performed. Moreover, truly long-distance $SU(3)$ breaking effects might not be captured in the short distance analysis. For example, a contribution from a light quark resonance with $m_R \approx m_D$ would not be captured in this analysis. For a sufficiently narrow resonance, this provides a mechanism for breaking of local quark-hadron duality ¹³.

An alternative way of estimating x and y is to start from the long distance contributions generated by the intermediate *hadronic* states. They arise from

the decays to intermediate states common to both D^0 and \bar{D}^0 . Therefore, a sum over all possible n -particle intermediate states allowed by the corresponding quantum numbers should be taken into account in Eq. (3). In practice, only a few states are considered, so only an order-of-magnitude estimate is possible. Even with this restriction, it is extremely difficult to reliably determine the total effect from a given subset of intermediate states due to the many decay modes with unknown final state interaction (FSI) phases. In addition, hadronic intermediate states in $D^0 - \bar{D}^0$ mixing are expected to occur as $SU(3)$ flavor multiplets, so there are cancelations among different contributions to x and y from the same multiplet. These flavor $SU(3)$ relations can be analyzed. The initial D state is an $SU(3)$ triplet, $D_i = (D^0, D^+, D_s^+)$, while the final state consists of a number of particles belonging to the octet representation,

$$M_k^i = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}. \quad (17)$$

A set of relations for the transition amplitudes $A_I = \langle D^0 | \mathcal{H}_W^{\Delta C=1} | I \rangle$ can be written. The effective Hamiltonian for D transitions, $\mathcal{H}_W^{\Delta C=1} \sim (\bar{\psi}c)(\bar{u}\psi)$ with $\psi = s, d$ transforms as $\bar{15} \oplus 6 \oplus 3 \oplus 3$ under $SU(3)_F$. Thus, D_i and M_k^i should be contracted with the vector $H(3)^i$ (3 Hamiltonian), antisymmetric (wrt upper indices) tensor $H(6)_k^{ij}$ (6 Hamiltonian), or symmetric tensor $H(\bar{15})_k^{ij}$ ($\bar{15}$ Hamiltonian). The $SU(3)$ relations for $\Delta\Gamma_D$ follow as $\Delta\Gamma_D \sim \langle D^0 | \mathcal{H}_W^{\Delta C=1} | I \rangle \langle I | \mathcal{H}_W^{\Delta C=1} | \bar{D}^0 \rangle$ and are rather complicated for a generic multi-particle intermediate state.

Let us elaborate on the simplest possible contribution, due to intermediate single-particle states¹³. These are rather simple to analyze, as the number of such intermediate states is constrained. A contribution to y from a resonance state R can be written as

$$y \Big|_{res} = \frac{1}{2\Gamma m_D} \sum_R \text{Im} \frac{\langle D_2 | \mathcal{H}_W | R \rangle \langle R | \mathcal{H}_W^\dagger | D_2 \rangle}{m_D^2 - m_R^2 + i\Gamma_R m_D} - (D_2 \leftrightarrow D_1). \quad (18)$$

The pseudoscalar 0^{-+} (scalar 0^{++}) intermediate states have $CP = -1$ ($CP = +1$) and contribute (in the CP-limit) to the D_1 (D_2) part of the above equation. In principle, this contribution exhibits a resonant enhancement for a narrow resonance with $m_R \approx m_D$. In reality, light quark states with such large masses are not narrow.

In the limit of degenerate s and d quark masses the contribution from the entire $SU(3)$ multiplet would vanish, as expected from the GIM cancelation

mechanism. Yet, $SU(3)$ is known to be badly broken in D -decays^{15,16}, so a sizable value for the width difference might not be surprising.

A set of $SU(3)$ relations for the $D \rightarrow R$ transitions follow from the following transition amplitude

$$A(D \rightarrow R) = A_3 D_i M_k^i H(\bar{3})^k + A_6 D_i H(6)^{ik} M_k^l + A_{15} D_i H(\bar{15})_l^{ik} M_k^l \quad (19)$$

A contribution of the octet of pseudoscalar single-particle intermediate states π_H , K_H , \bar{K}_H , η_H (and possibly η'_H with $\eta_H - \eta'_H$ mixing angle θ_H) is

$$y|_{\text{octet}}^{\text{res}} = y^{(K_H)} - \frac{1}{4} y^{(\pi_H)} - \frac{3 \cos^2 \theta_H}{4} y^{(\eta_H)} - \frac{1 \sin^2 \theta_H}{4} y^{(\eta'_H)} \quad , \quad (20)$$

with the mixing amplitudes induced by resonance R calculated to be

$$y^{\text{res}} = -\frac{|H_R|^2}{m_D^3 \Gamma} \frac{\gamma_R}{(1 - \mu_R)^2 + \gamma_R^2} \quad , \quad (21)$$

where $|H_R|^2 \equiv \langle D^0 | \mathcal{H}_W | R \rangle \langle R | \mathcal{H}_W^\dagger | \bar{D}^0 \rangle$, and the dimensionless quantities $\mu_R \equiv m_R^2/m_D^2$ and $\gamma_R \equiv \Gamma_R/m_D$ are the reduced squared-mass and width of the resonance.

No reliable information about the size of $\langle D | \mathcal{H}_W | R \rangle$ matrix elements is available at the moment. A typical contribution to y from one 0^{-+} single-particle heavy intermediate state can be calculated using vacuum insertion ansatz. This implies $|H_R|^2 = \mu_R f_R^2 m_D (G_F a_2 f_D \xi_d / \sqrt{2})^2$, with f_R being the resonance decay constant. Making an “educated guess” about the size of f_R , it can be shown that a typical contribution from a 0^{-+} amounts to a *few* $\times 10^{-4}$ (see Ref.¹³), but might be larger.

An estimate of H_R for a 0^{++} single-particle heavy intermediate state (like $K^*(1430)$ or $K^*(1940)$) can be obtained using the soft pion theorem arguments of Ref.¹⁴ and measured branching ratios for $D^+ \rightarrow R\pi^+$ transitions. Assuming that expected corrections to the soft pion theorem are not large we derive for $R = K^*(1430)$

$$y_{0^{++}} = -\tan^2 \theta_C \frac{8\pi f_\pi^2}{q_\pi} \frac{\mathcal{B}(D^+ \rightarrow K^*(1430)\pi^+)}{f m_D^2} \frac{\Gamma_{D^+}}{\Gamma_{D^0}} \frac{\gamma_R}{(1 - \mu_R)^2 + \gamma_R^2} \quad , \quad (22)$$

where $q_\pi = 0.368 \text{ GeV}$ is a pion’s momentum, $\mathcal{B}(D^+ \rightarrow K^*(1430)\pi^+) \simeq 0.023$, $f \equiv \mathcal{B}(K^*(1430) \rightarrow K\pi) \simeq 0.62$, $f_\pi = 0.13 \text{ GeV}$ is a pion’s decay constant, and $\Gamma_{D^+}/\Gamma_{D^0} \simeq 0.4$. This gives

$$|y_{0^{++}}(1430)| \simeq 0.02\% \quad , \quad (23)$$

which is in the same ballpark as y_{0-+} . Now, if we *assume* that $H_{K^*(1430)} \simeq H_{K^*(1940)}$,

$$|y_{0++}(1940)| \simeq 0.1\%, \quad (24)$$

It is clear from the Eq.(20) that $y = 0$ in the $SU(3)_F$ limit, where $\mu_i = \mu_0, \gamma_i = \gamma_0$, and $H_i = H_0$ for $i = \pi_H, K_H, \eta_H$ and $\sin^2 \theta_H \rightarrow 0$. It is therefore necessary to assess the pattern of $SU(3)$ -symmetry breaking in Eq. (20). Neglecting singlet-octet mixing and assuming that

$$\begin{aligned} \mu_i &= \mu_0 + \delta\mu_i, \\ \gamma_i &= \gamma_0 + \delta\gamma_i \\ |H_i| &= |H_0| + \delta H_i, \end{aligned} \quad (25)$$

we obtain an estimate of y

$$\begin{aligned} -y|_{\text{octet}}^{\text{res}} \times m_D^3 \Gamma \frac{(1-\mu_0)^2 + \gamma_0^2}{|H_0|\gamma_0} &= 2 \frac{\mu_0(1-\mu_0)}{(1-\mu_0)^2 + \gamma_0^2} \left[\frac{\delta\mu_K}{\mu_0} - \frac{1}{4} \frac{\delta\mu_\pi}{\mu_0} - \frac{3}{4} \frac{\delta\mu_\eta}{\mu_0} \right] \\ &+ \frac{(1-\mu_0)^2 - \gamma_0^2}{(1-\mu_0)^2 + \gamma_0^2} \left[\frac{\delta\gamma_K}{\gamma_0} - \frac{1}{4} \frac{\delta\gamma_\pi}{\gamma_0} - \frac{3}{4} \frac{\delta\gamma_\eta}{\gamma_0} \right] \\ &+ 2 \left[\frac{\delta H_K}{|H_0|} - \frac{1}{4} \frac{\delta H_\pi}{|H_0|} - \frac{3}{4} \frac{\delta H_\eta}{|H_0|} \right]. \end{aligned} \quad (26)$$

Unfortunately, many of the parameters of Eq. (26) are not known. Yet, it's not unlikely that the total resonance contribution could amount to $y \approx 0.1\%$ or so.

Let us briefly discuss a contribution from charged pseudoscalar *two-body* intermediate state. It was originally considered in Refs. ^{5,17,18} and estimated to be potentially large,

$$y_2 = \frac{1}{2m_D\Gamma} \sum_{p_1, p_2} \text{Re} \int \frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1} \frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2} \langle D^0 | \mathcal{H}_W | p_1, p_2 \rangle \langle p_1, p_2 | \mathcal{H}_W^\dagger | \bar{D}^0 \rangle \quad (27)$$

where one must sum over all intermediate state particles p_1, p_2 , not only ground state mesons. For the charged pseudoscalar state $\{p_1, p_2\} = \{K^+, K^-\}, \{\pi^+, \pi^-\}, \{K^+, \pi^-\}$, and $\{K^-, \pi^+\}$. As before, the $SU(3)$ relations among amplitudes imply cancelations. These cancelations occur within each multiplet, however broken $SU(3)$ assures that they are not complete. Residual contributions from each multiplet then have to be summed up.

In some cases available experimental data can be used. For example, for $p_1, p_2 = K^+ K^-$ we easily obtain from Eq. (27) that $y_{KK} = \mathcal{B}(D^0 \rightarrow K^+ K^-)$,

which is well measured. Thus, the charged pseudoscalar contribution can be easily estimated

$$y_2 = (5.76 - 5.29 \cos \delta) \times 10^{-3}, \quad (28)$$

where the strong phase difference δ is defined in Eq. (7). Taking $-1 < \cos \delta < 0$ (see discussion in ^{15,20}) implies that

$$0.6 \times 10^{-2} < y_2 < 1.1 \times 10^{-2} \quad (29)$$

or

$$y_2 < 1.53 \times 10^{-3}, \quad (30)$$

if $\delta < 40^\circ$, as favored by hadronic models ¹⁵. Unfortunately, the experimental information about many other relevant hadronic decays is not available, so model-dependence of the final result is unavoidable.

We have to note, however, that phase space effects should profoundly distort the patterns of GIM cancelations for the intermediate states containing excited mesons ¹². For example, let us take the decay modes with one ground state and one excited state (first radial excitation) mesons, like $K(1460)$ or $\pi(1300)$. Clearly, the final state $K(1460)K$ is kinematically forbidden, while other decays in the same $SU(3)$ multiplet are not! Unfortunately, no experimental data exists for these transitions.

To summarize our discussion, we note that it is quite likely theoretically that $y \sim 0.1\%$, as it is dominated by a SM $\Delta C = 1$ contribution, whereas x can be as large as a percent in certain extensions of the Standard Model. Some long-distance contributions to y can also be as large as a percent, but they are either canceled by similar contribution from the same $SU(3)$ multiplet or require values of strong phases that are disfavored by $SU(3)$ and hadronic models.

3 Experimental situation

There are two intriguing experimental measurements providing some information about $D\bar{D}$ mixing parameters. The FOCUS experiment fits the time dependent decay rates of the singly-Cabibbo suppressed and the Cabibbo-favored modes to pure exponentials. We define $\hat{\Gamma}$ to be the parameter that is extracted in this way. More explicitly, for a time dependent decay rate with $\Gamma[D(t) \rightarrow f] \propto e^{-\Gamma t}(1 - z\Gamma t + \dots)$, where $|z| \ll 1$, we have $\hat{\Gamma}(D \rightarrow f) = \Gamma(1+z)$. The above equations imply the following relations:

$$\begin{aligned} \hat{\Gamma}(D^0 \rightarrow K^+ K^-) &= \Gamma [1 + R_m(y \cos \phi - x \sin \phi)], \\ \hat{\Gamma}(\bar{D}^0 \rightarrow K^+ K^-) &= \Gamma [1 + R_m^{-1}(y \cos \phi + x \sin \phi)], \\ \hat{\Gamma}(D^0 \rightarrow K^- \pi^+) &= \hat{\Gamma}(\bar{D}^0 \rightarrow K^+ \pi^-) = \Gamma. \end{aligned} \quad (31)$$

Note that deviations of $\hat{\Gamma}(D \rightarrow K^+K^-)$ from Γ do not require that $y \neq 0$. They can in principle be accounted for by $x \neq 0$ and $\sin \phi \neq 0$, but then they have a different sign in the D^0 and \overline{D}^0 decays. FOCUS combines the two $D \rightarrow K^+K^-$ modes. To understand the consequences of such an analysis, one has to consider the relative weight of D^0 and \overline{D}^0 in the sample²⁰. Let us define A_{prod} as the production asymmetry of D^0 and \overline{D}^0 , $A_{\text{prod}} \equiv (N(D^0) - N(\overline{D}^0))/(N(D^0) + N(\overline{D}^0))$. Then, if A_{prod} is small (as suggested by E687 data) and if $R_m^{\pm 2} = 1 \pm A_m$, with A_m being small (as suggested by CLEO),

$$\begin{aligned} y_{\text{CP}} &\equiv \frac{\hat{\Gamma}(D \rightarrow K^+K^-)}{\hat{\Gamma}(D^0 \rightarrow K^-\pi^+)} - 1 \\ &= y \cos \phi - x \sin \phi \left(\frac{A_m}{2} + A_{\text{prod}} \right). \end{aligned} \quad (32)$$

The one sigma range measured by FOCUS is

$$y_{\text{CP}} = (3.42 \pm 1.57) \times 10^{-2}. \quad (33)$$

The CLEO measurement gives the coefficient of each of the three terms (1 , Γt and $(\Gamma t)^2$) in the doubly-Cabibbo suppressed decays. Such measurements allow a fit to the parameters R , R_m , $x' \sin \phi$, $y' \cos \phi$, and $x^2 + y^2$. CLEO quotes the following one sigma ranges:

$$\begin{aligned} R &= (0.48 \pm 0.13) \times 10^{-2}, \\ y' \cos \phi &= (-2.5^{+1.4}_{-1.6}) \times 10^{-2}, \\ x' &= (0.0 \pm 1.5) \times 10^{-2}, \\ A_m &= 0.23^{+0.63}_{-0.80}. \end{aligned} \quad (34)$$

As we shall see shortly, a combination of FOCUS (33) and CLEO (34) results provides powerful constraints on the values of $D^0 - \overline{D}^0$ mixing parameters.

4 Interpretation and Conclusions

Let us now see the implications of the new CLEO and FOCUS measurements for the value of y . *We shall assume that the true values of the mixing parameters are within one sigma of the results provided by these two experiments.* First of all, based on the available bounds on x , $\sin \phi$ and $|A_m|$ ²⁰, one can argue that it is very unlikely that FOCUS result is accounted for by the second term in Eq. (32). Therefore, *if the true values of the mixing parameters are within the one sigma ranges of CLEO and FOCUS measurements, then y is of order of a (few) percent.* More specifically, $y \cos \phi \approx 0.034 \pm 0.016$! This is a

rather surprising result (see Section 2). Also, if CLEO and FOCUS results are consistent, then

$$\begin{aligned}\cos \delta - (x/y) \sin \delta &= -0.73 \pm 0.55, \text{ or} \\ \cos \delta &\sim +0.65 \text{ if } |x| \sim |y| \\ \cos \delta &\sim -0.18 \text{ if } |x| \ll |y|\end{aligned}\tag{35}$$

which leads to another interesting conclusion: *if the true values of the mixing parameters are within the one sigma ranges of CLEO and FOCUS measurements, then the difference in strong phases between the $D^0 \rightarrow K^+\pi^-$ and $D^0 \rightarrow K^-\pi^+$ decays is very large.*

Since the strong phase δ vanishes in the $SU(3)$ flavor symmetry limit, the result of Eq. (35) is also rather surprising (for a discussion of the strong phase difference in $D \rightarrow K\pi$ see^{15,21}). On the other hand, there are other known examples of $SU(3)$ breaking effects of order one in D decays, so perhaps we should not be prejudiced against a very large δ .

Charm physics experiments have started to probe an interesting region of $D^0 - \bar{D}^0$ mixing parameter space, therefore new and excited results from the existing and future²² experiments are warranted.

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References

1. H. Georgi, Phys. Lett., **B297** (1992) 353.
2. T. Ohl *et al.*, Nucl. Phys. **B403** (1993) 605.
3. A. A. Petrov, Phys. Rev. **D56** (1997) 1685.
4. A. Datta and M. Khambakhar, Z. Phys., **C27** (1985) 515.
5. J. F. Donoghue, E. Golowich, B. Holstein and J. Trampetic, Phys. Rev., **D33** (1986) 179.
6. See, for example, J. F. Donoghue, E. Golowich and B. R. Holstein, "Dynamics of the standard model," *Cambridge Univ. Press* (1992).
7. J. M. Link *et al.* [FOCUS Collaboration], hep-ex/0004034.
8. R. Godang *et al.* [CLEO Collaboration], hep-ex/0001060.
9. P. F. Harrison and H. R. Quinn [BABAR Collaboration], *The BaBar physics book: Physics at an asymmetric B factory*, SLAC-R-0504, 1998.
10. E. Golowich and A. A. Petrov, unpublished; E. Golowich, talk at the Workshop on CP Violation, Adelaide, Australia, July 3 - 8, 1998.
11. I. I. Bigi and N. G. Uraltsev, hep-ph/0005089.
12. Y. Grossman, A. F. Falk, Z. Ligeti and A. A. Petrov, to be published

13. E. Golowich and A. A. Petrov, Phys. Lett. **B427**, 172 (1998) [hep-ph/9802291].
14. M. Gronau, Phys. Rev. Lett. **83**, 4005 (1999) [hep-ph/9908237].
15. A. F. Falk, Y. Nir and A. A. Petrov, JHEP **9912**, 019 (1999) [hep-ph/9911369].
16. L.-L. Chau and H.-Y. Cheng, Phys. Lett. **B333** (1994) 514.
17. L. Wolfenstein, Phys. Lett., **B164** (1985) 170.
18. F. Buccella, M. Lusignoli and A. Pugliese, Phys. Lett., **B379** (1996) 249.
19. *E.g.*, see G. Blaylock, A. Seiden and Y. Nir, Phys. Lett., **B355** (1995) 555.
20. S. Bergmann, Y. Grossman, Z. Ligeti, Y. Nir and A. A. Petrov, Phys. Lett. **B486**, 418 (2000) [hep-ph/0005181].
21. A. A. Petrov, talk at 1999 Chicago Conference on Kaon Physics (K 99), Chicago, IL, 21-26 Jun 1999 [hep-ph/9909312].
22. A. A. Petrov, talk at 23rd Johns Hopkins Workshop on Current Problems in Particle Theory: Neutrinos in the Next Millennium, Baltimore, Maryland, 10-12 Jun 1999 [hep-ph/0003026].